

# "Integrable systems and automorphic forms"

## (Lille, May 13-16, 2024)

*Les exposés auront lieu dans l'Amphi Turing au M3*

### Monday, May 13

- 9:30-10:30 E. Ferapontov "Integrable Lagrangians, modular forms and degenerations"
- 11:00-12:00 D. Dinh "Separation of variables of the Hitchin system: a classical limit of geometric Langlands for  $SL_2(\mathbb{C})$ "
- 14:30-15:30 C. Oelen "Automorphic Lie algebras on complex tori"
- 15:40-16:40 A. Moreau "On a series of simple affine VOAs at non-admissible level arising from rank one 4D SCFTs"
- 17:10-18:10 J. Fasquel "Connecting W-algebras and their representations"

### Tuesday, May 14

- 9:30-10:30 I. Bobrova "Affine Weyl groups and non-abelian discrete systems"
- 11:00-12:00 S. Opanasenko "Bi-Hamiltonian geometry of WDVV equations"
- 14:30-15:30 I. Biswas "Vector bundles and connections on Riemann surfaces with projective structure"
- 15:40-16:40 G. Comaschi "Instantons on Contact Fano Manifolds"
- 17:10-18:10 R. Terpereau "Equivariant vector bundles on Fano varieties"

### Wednesday, May 15

- 9:30-10:30 W. Nahm "A mathematically natural axiom for Quantum Field Theory"
- 11:00-12:00 D. Israël "On the dressed elliptic genus for non-Kähler manifolds of Fu-Yau type"
- 14:30-15:30 A. Sarti "Complex Reflection Groups, K3 surfaces and Lehrer-Springer theory"
- 15:40-16:40 V. Gritsenko "Irrationality of the moduli spaces of polarised generalised Kummer varieties"
- 17:10-18:10 S. Hohenegger "Non-perturbative Symmetries of Little Strings and Affine Quiver Algebras"

### Thursday, May 16

- 9:30-10:30 V. Roubtsov "Higher Bessel functions 1 year later"
- 11:00-12:00 A. Odesski "When the Fourier transform is one loop exact?"

# Abstracts

**Indranil Biswas (TIFR, Mumbai)** "Vector bundles and connections on Riemann surfaces with projective" structure". (With Jacques Hurtubise and Vladimir Roubtsov.)

**Abstract:** Let  $B_g(r)$  be the moduli space of stable bundles of rank  $r$  and degree 0 on a Riemann surface. We construct a  $T^*B_g(r)$ -torsor  $H_g(r)$ . It is shown that  $H_g(r)$  has a holomorphic symplectic structure compatible with the  $T^*B_g(r)$ -torsor structure. We also describe  $H_g(r)$  in terms of the second order matrix valued differential operators. It is shown that  $H_g(r)$  is identified with the  $T^*B_g(r)$ -torsor given by the sheaf of holomorphic connections on the theta line bundle over  $B_g(r)$ .

**Irina Bobrova (Max-Planck-Institut für Mathematik in den Naturwissenschaften, Leipzig)**  
"Affine Weyl groups and non-abelian discrete systems"

**Gaia Comaschi (Université de Pau)** "Instantons on Contact Fano Manifolds"

**Duong Dinh (Max-Planck-Institut für Mathematik, Bonn)** "Separation of variables of the Hitchin system: a classical limit of geometric Langlands for  $SL_2(\mathbf{C})$ "

**Justine Fasquel (University of Melbourne)** "Connecting  $W$ -algebras and their representations"

**Abstract:**  $W$ -algebras form a large family of vertex algebras associated to nilpotent orbits of simple Lie algebras. They provide an additional framework to study the representations of certain infinite dimensional Lie algebras. Indeed,  $W$ -algebras can be obtained from the latter by applying certain quantized Hamiltonian reductions. It is believed that most of the time the reduction procedure can be reversed to reconstruct the representations of the underlying affine Lie algebra.

In this talk, we will discuss inverse reductions as well as their consequences on the representation theory based on examples in small ranks. The talk reports on a recent paper with T. Creutzig, A. Linshaw and N. Nakatsuka and on going work with C. Raymond and D. Ridout.

**Evgeny Ferapontov (University of Loughborough)** "Integrable Lagrangians, modular forms and degenerations"

**Valery Gritsenko (Université de Lille & NRU HSE)** "Irrationality of the moduli spaces of polarised generalised Kummer varieties"

**Stefan Hohenegger (Institut National de physique nucléaire et de physique des particules, Lyon)** "Non-perturbative Symmetries of Little Strings and Affine Quiver Algebras"

**Dan Israël (LPTHE, Université de Sorbonne)** "On the dressed elliptic genus for non-Kähler manifolds of Fu-Yau type"

**Anne Moreau (Université Paris-Saclay)** "On a series of simple affine VOAs at non-admissible level arising from rank One 4D SCFTs"

**Werner Nahm (IAS Dublin)** "A mathematically natural axiom for Quantum Field Theory"

**Abstract:** Euclidean quantum field theory can be defined as the theory of continuous functors from a category of manifolds to a category of self-dual real vector spaces, with rather obvious monoidal and duality properties. In this approach it is straightforward to define fields and their scaling dimensions and to relate them to deformations of the functor. In a second step the transition to Minkowskian theories and their integrability properties will be discussed.

**Alexandre Odesski (Brock University)** "When the Fourier transform is one loop exact?"

**Abstract:** We investigate the question: for which functions  $f(x_1, \dots, x_n), g(x_1, \dots, x_n)$  the asymptotic expansion of the integral

$$\int g(x_1, \dots, x_n) e^{\frac{f(x_1, \dots, x_n) + x_1 y_1 + \dots + x_n y_n}{\hbar}} dx_1 \dots dx_n$$

consists only of the first term. We reveal a hidden projective invariance of the problem which establishes its relation with geometry of projective hypersurfaces of the form  $\{(1 : x_1 : \dots : x_n : f)\}$ . We also construct various examples, in particular we prove that Kummer surface in  $\mathbb{P}^3$  gives a solution to our problem. This is a joint paper with Maxim Kontsevich.

**Casper Oelen (Heriot-Watt University, Edinburgh)** "Automorphic Lie algebras on complex tori"

**Stanislav Opanasenko (Università del Salento, Lecce)** "Bi-Hamiltonian geometry of WDVV equations"

**Vladimir Roubtsov (Université d'Angers)** "Higher Bessel functions 1 year later"

**Abstract:** We revise, upgrade and correct our results reported in Lille Workshop-Mai 2023. We consider the generating function  $\Phi^{(N)}$  for the reciprocals  $N$ -th power of factorials. We show a connection of product formulas for such series with the periods for certain families of algebraic hypersurfaces. We describe the singular loci of these surfaces and show that they are given by zeros of the Buchstaber-Rees polynomials, entering in  $N$ -valued group laws. We propose a generalized

Frobenius method and use it to obtain special expansions for multiplication kernels in the sense of Kontsevich. Using these expansions we provide some experimental results in order to connect  $N$ -Bessel kernels and the hierarchies of the palindromic unimodal polynomials. We study the properties of such polynomials and conjecture positivity of their roots.

The results are based on a joint work with I. Gaiur (Toronto) and D. van Straten (Mainz).

**Alessandra Sarti (Université de Poitiers)** "Complex Reflection Groups, K3 surfaces and Lehrer-Springer theory"

**Abstract:** I will talk about a long project in collaboration with Cédric Bonnafé, that relates K3 surfaces and complex reflection groups. This generalizes and explains some results of 2003 by W. Barth and myself. In fact by using complex reflection groups and Lehrer-Springer theory we obtained the following three main results :

1. Classification of some finite groups of maximum order acting on K3 surfaces.
2. Classification of all K3 surfaces that one can obtain as quotient of surfaces in complex projective three space by certain subgroups of finite complex reflection groups of rank four.
3. Description of elliptic fibrations on the previous K3 surfaces. After an overview of the results, I will explain more in detail point 2, in particular I will introduce some Lehrer-Springer theory from the theory of complex reflection groups, which is a fundamental tool to avoid a case-by-case analysis in the classification.

**Ronan Terpereau (Université de Lille)** "Equivariant vector bundles on Fano varieties"

**Abstract:** This talk addresses the problem of constructing homogeneous vector bundles, i.e. vector bundles invariant under the action of an algebraic group, in which the algebraic group acts on the base of the bundle with an open orbit. Producing such bundles is generally difficult, but we'll see how instanton bundles, which appeared in the 1970s to explain fundamental forces in physics, offer an approach for constructing them when the base of the bundle is a Fano variety. This is a joint work with Daniele Faenzi.