Université Lille Nord de France
Pôle de Recherche et d'Enseignement Supérieur

Ecole Graduée 631 MADIS

Sujet de thèse en Mathématique proposé en 2023

Titre : Statistical properties of equilibrium measures in several complex variables

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Descriptif :

The goal of this project is to study statistical properties of natural invariant measures for dynamical systems, mainly in several complex variables.

Given a polynomial map f on C, one can decompose the plane in two subsets, one where the dynamics is simple (the Fatou set), and one where the dynamics is chaotic (the Julia set). Studying the orbits for points in the Julia set is, essentially by definition, an impossible task. One can then take a probabilistic approach. Given a real function on C and an invariant measure \nu, one can study the sequence of random variables u \circ f^n with respect to \nu. As \nu is invariant, they have the same distribution. On the other hand, precisely because they arise from a deterministic system, they are not independent. A main question in dynamics is to study this sequence, and show that it behaves as close as possible to a sequence of i.i.d. random variables, for instance proving a Central Limit Theorem for such sequence.

Several approches to this problem have been proposed in a number of contexts. Essentially, most of them rely on some hyperbolicity assumption on the dynamical system. Recently, F. Bianchi and T.-C. Dinh developed a new general approach to the problem, also valid in any dimension, and with no need of hyperbolicity assumptions. Thanks to this approach, they studied in detail the case of endomorphisms of projective spaces in any dimension.

This project is a continuation of that work. A first question is to generalize the method above to the (infinitely dimensional and non algebraic) class of polynomial-like maps in any dimension. These maps are geometrically expanding, but not uniformly hyperbolic. A second step is to consider systems of saddle type (i.e., displaying both attracting and repelling directions), possibly starting with some special cases.