

# The relaxation of the Cahn-Hilliard equation for the modelling of tumors and its numerical simulation

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The Cahn-Hilliard equation, arising from physics, describes the phase separation occurring in a material during a sudden cooling process and is the subject of many pieces of research [2]. An interesting application of this equation is its capacity to model cell populations undergoing attraction and repulsion effects [4]. For this application, we consider a variant of the Cahn-Hilliard equation with a single-well potential and a degenerate mobility [1]. This particular form introduces numerous difficulties especially for numerical simulations. We propose a relaxation of the equation to tackle these issues and analyze the resulting system [3]. Interestingly, this relaxed version of the degenerate Cahn-Hilliard equation bears some similarity with a nonlinear Keller-Segel model. We also describe a simple finite element scheme that preserves the critical physical (or biological) properties using an upwind approach.

## References

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