

ECOLE DOCTORALE 631 MADIS

SUJET DE THÈSE EN MATHÉMATIQUE PROPOSÉ EN 2024

Title : Image of analytic map germs

Directeur de thèse : Mihai Tibăr — E-mail : mtibar@univ-lille.fr

Co-directeur de thèse : Cezar Joița, IMAR, Roumanie — E-mail : cezar.joita@imar.ro

Laboratoire : Painlevé

Equipe : GT

A grant for 22 months may cover the first part of this PhD thesis, as part of the project “[Singularities and Application](#)” PNRR-III-C9-2023-I8 - Project CF 132/31.07.2023, founded by the European Union - NextGenerationEU - through Romania’s National Recovery and Resilience Plan, based at [IMAR](#).

Let $F : (X, x) \rightarrow (Y, y)$ be an analytic map germ, where (X, x) and (Y, y) are germs of real or complex analytic varieties. The need of assuming that the image of an analytic map germ is well-defined *as a set germ* has been pointed out by John Mather in his fundamental study on the stratified structure of maps [Ma]. This condition was recently studied in [JT1] and [JT2]. One of the motivations for its study was the characterisation of the situation when the image is locally open. In case of holomorphic maps, such a characterisation was conjectured by Huckleberry [Hu], and was proved in [JT2]. This problem is totally different from the classical Open Mapping Theorem in complex analysis.

In very simple examples the image of F is however not well-defined as a set germ, as pointed out in [Hu] and [JT2]. One may then ask if there is a good replacement for the image in such cases. This question is studied in [JT3] in the case of holomorphic map germs with 2-dimensional target.

Here are a few directions of study that might be addressed in the thesis:

- Find the structure of the local image of a holomorphic map germ with higher dimensional target, for instance $F : (X, x) \rightarrow (\mathbb{C}^3, 0)$.
- Study the similar image problem in the case of real analytic map germs.
- Define a notion of fibration for map germs which do not have a well-defined image as a set germ, and find sufficient conditions for the existence of such a fibration, with applications.

These features would lead to non-trivial extensions of the well-known results, the classical ones by Milnor [Mi], Lê [Le], and the recent ones [ART], to the case of map germs with non well-defined images as set germs.

REFERENCES

- [ART] R.N. Araújo dos Santos, M. Ribeiro, M. Tibăr, *Fibrations of highly singular map germs*, Bull. Sci. Math. 155 (2019), 92-111.
- [Hu] A.T. Huckleberry, *On local images of holomorphic mappings*. Ann. Sc. Norm. Sup. Pisa (3) 25 (1971), 447-467.
- [JT1] C. Joița, M. Tibăr, *Images of analytic map germs and singular fibrations*. Eur. J. Math. 6 (2020), no 3, 888-904.
- [JT2] C. Joița, M. Tibăr, *Images of analytic map germs*. Arkiv för Matematik 59 (2021), 345-358.
- [JT3] C. Joița, M. Tibăr, *What is the local image of an analytic map?*, manuscript.
- [Le] Lê D.T., *Complex analytic functions with isolated singularities*. J. Algebraic Geom. 1 (1992), 83-100.
- [Ma] J.N. Mather, *Stratifications and mappings*. Dynamical systems (Proc. Sympos., Univ. Bahia, Salvador, 1971), pp. 195-232. Academic Press, New York, 1973.
- [Mi] J.W. Milnor, *Singular points of complex hypersurfaces*. Annals of Mathematics Studies, No. 61 Princeton University Press, Princeton, N.J.; University of Tokyo Press, Tokyo 1968.