Ecole Doctorale 631 MADIS

Sujet de thèse en Mathématique proposé en 2024

Titre : Comparing witnesses of nonclassicality in quantum mechanics

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Descriptif :

The idea that it should be possible to exploit the particular features of quantum systems in order to obtain a ``quantum advantage" has gained considerable ground in recent years and much effort has gone into all aspects of this basic tenet, theoretically, experimentally and, more recently, technologically. The goal is to exploit those properties that most markedly distinguish quantum systems from classical ones to vastly improve various procedures and protocols in computing, cryptography, communication, metrology and simulation.

A central question in this context is the identification of the classical-quantum boundary in a system's state space. Which are the states that can or cannot be hoped to provide such a quantum advantage?
One of the tools that have proven instrumental in this questioning are quasi-probability distributions. For the continuous variable theories, the Wigner function is central, as well as the Glauber-Sudarshan P-function. Their negativity is a hallmark of non-classicality and a necessary condition to obtain a possible quantum advantage. In finite dimension, Kirkwood-Dirac KD) distributions have very recently come to the forefront in a variety of contexts. In fact, given any pair of non-commuting observables, one can associate a KD-distribution to any quantum state. Again non-positivity is a prerequisite here to obtain a quantum advantage.

In both cases, a number of witnesses, measures and monotones have been designed to determine if the quasi-probability of given state does or does not manifest non-positivity and to assess the degree to which such non-positivity is present. It is the goal of this thesis project to compare the existing such measures through upper/lower bounds and to thus better understand their meaning and role.

No prior knowledge of quantum mechanics is required, but an interest in the application of mathematics to physics is expected. The mathematics involved in this project is mostly functional analysis, Hilbert space theory (operators, spectral theory) and probability theory. The capability of testing conjectures with numerical simulations is a plus.